This PhD thesis proposes divergence-free Discontinuous Galerkin formulations providing high orders of accuracy for incompressible viscous flows.

A new Interior Penalty Discontinuous Galerkin (IPM-DG) formulation is developed, leading to a symmetric and coercive bilinear weak form for the diffusion term, and achieving high-order spatial approximations. It is applied to the solution of the Stokes and Navier-Stokes equations. The velocity approximation space is decomposed in every element into a solenoidal part and an irrotational part. This allows to split the IPM weak form in two uncoupled problems. The first one solves for velocity and hybrid pressure, and the second one allows the evaluation of pressures in the interior of the elements. This results in an important reduction of the total number of degrees of freedom for both velocity and pressure.

The introduction of an extra penalty parameter leads to an alternative DG formulation for the computation of solenoidal velocities with no presence of pressure terms. Pressure can then be computed as a post-process of the velocity solution. Other DG formulations, such as the Compact Discontinuous Galerkin method, are contemplated and compared to IPM-DG.

High-order Implicit Runge-Kutta methods are then proposed to solve transient incompressible problems, allowing to obtain unconditionally stable schemes with high orders of accuracy in time. For this purpose, the unsteady incompressible Navier-Stokes equations are interpreted as a system of Differential Algebraic Equations, that is, a system of ordinary differential equations corresponding to the conservation of momentum equation, plus algebraic constraints corresponding to the incompressibility condition.

Numerical examples demonstrate the applicability of the proposed methodologies and compare their efficiency and accuracy.